MAE 427
FM1

Measurement of Viscosity and Turbulence Transition

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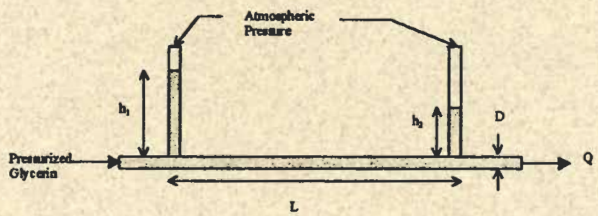
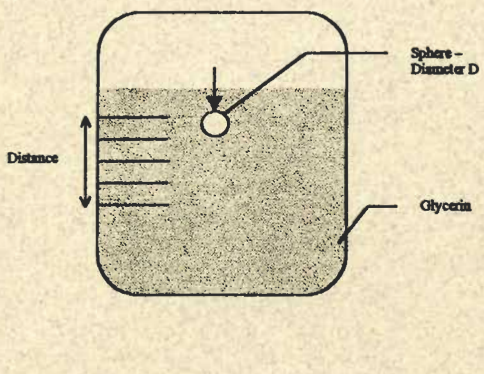
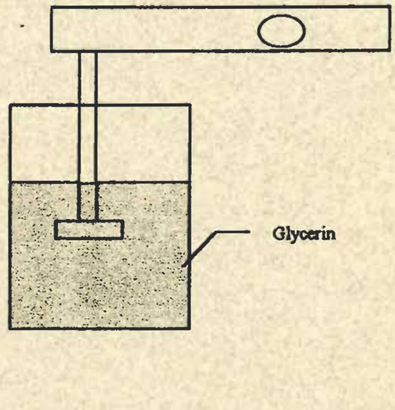
Fall 1998
Lab Section 10







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FANTASTIC W

The presentation

marvellou

Summary

The goals of this experiment were to:

1) Determine the viscosity of a Newtonian fluid (pure glycerin) by three methods
   - By measuring the pressure drop and flow rate in a smooth pipe of know diameter after determining that the flow is laminar
   - By measuring the time it takes a small sphere to fall a known distance through the fluid
   - By measuring the torque required to rotate a spindle immersed in the fluid at a given speed, using a commercial viscosity meter.
2) Study the transition from laminar to turbulent flow in a pipe

The results of this experiment were:

Part 1)

Laminar Pipe Flow	Sphere in Stokes Flow	Viscosity Meter
μ_{avg} = 0.755 kg/m/s	μ_{avg} = 0.964 kg/m/s	μ_{avg} = 0.807 kg/m/s

Good idea to include this here!

If there had been no experimental error, the laminar pipe flow would have given the most accurate estimation of the viscosity. It is based on an exact solution of the Navier-Stokes equation as opposed to the sphere in Stokes flow method, which is based on an approximation of the Navier-Stokes equation. The accuracy of the viscosity meter cannot fully be trusted since it depends on how well the meter was calibrated and we have no information regarding that. Considering the inevitability of experimental error, however, the sphere in Stokes flow method is the best way of determining the dynamic viscosity coefficient. The laminar pipe flow method is much more prone to experimental error and the data collected from the Stokes flow method can be modified to predict the viscosity very accurately.

Good!

Best estimation of μ: 0.925 kg/m/s

Part 2)

Critical Reynolds number: ≈2000

We observed the transition from laminar to turbulent flow in a pipe and found that it occurred at roughly 2000 Re. Turbulent flow is characterized by greater pressure gradients in the pipe, which arise from increased shear stresses in the fluid at the walls of the pipe. The Reynolds number is a ratio of the inertial forces in a fluid to the viscous forces. The transition from laminar flow to turbulent flow occurs because the inertial forces increase to a point where the viscous forces cannot dampen small fluctuations in the fluid flow and so mixing occurs. Our critical Reynolds number is very small compared to modern estimations, but our experimental setup induced many disturbances in the flow, which caused the transition to occur at a lower Reynolds number.

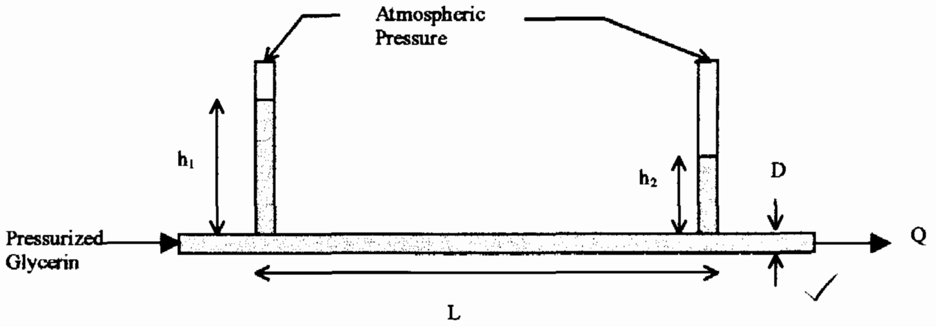
Very Good!

Method

Part 1 – Viscosity Measurement

Section1(i) – Viscosity from Pipe Flow Experiment

Apparatus:



Nice sketch!

Figure 1 – Smooth pipe with two manometers.

Theory:

Navier-Stokes Equation:

$$-\nabla p + \mathbf{B} + \mu\nabla^2\mathbf{u} = \rho[d\mathbf{u}/dt + (\mathbf{u}\nabla)\mathbf{u}]$$

Force = mass*acceleration

Exact Solution of the Navier-Stokes Equation for Laminar Flow in a Circular Pipe:

$$\Delta P = [128\mu LQ][\pi D^4]^{-1}$$

ΔP = change in pressure
μ = dynamic viscosity coefficient
L = length of the pipe over which the pressure drop is being measured
Q = volume flow rate
D = pipe diameter

Great use of equations and good definition of variables!

Calculation of ΔP from the experimental setup:

$$\Delta P = \rho g(h_1 - h_2)$$

ρ = glycerin density
g = gravitational constant
h_1 = height of the glycerin in manometer 1
h_2 = height of the glycerin in manometer 2

Calculation of μ from the collected data:

$$\mu = k(\pi/8)(R^4/L)$$

k = slope of the ΔP vs. Q least squares fit line

$R = (0.5)D$

Procedure:

The Navier-Stokes equation is analogous to Newton's 2nd Law of Motion for fluids. The mass of the fluid times its acceleration is equal to the sum of the forces. Writing the Navier-Stokes equation in one dimension and integrating it for the laminar flow pattern in a circular pipe yielded a relationship between the viscosity of the fluid and the change in pressure over a distance in the pipe. The viscosity of glycerin, therefore, can be found by measuring the pressure difference of the glycerin as it flows through a circular pipe over a known distance.

The apparatus for this section can be seen in Figure 1. It consists of a smooth round pipe of known diameter with two manometers attached to it a given distance apart. The manometers measure the pressure inside the pipe at the points where they are attached. The pressure is read from the manometer by noting the height that the fluid rises in the manometer when it is open to atmospheric pressure. The difference in the two manometer heights relates the difference in pressure over the length L.

The data for this section were gathered by observing the height difference in the manometers for several different flow rates. The flow rates were calculated by timing how long it took for the glycerin to flow from the pipe into a graduated beaker and the flow rates were controlled by a simple valve. k could then be found by graphing the difference in pressure versus the associated flow rate and fitting a straight line to the data points. The raw data can be found in Appendix A.

Section 1(ii) – Sphere in Stokes Flow

Apparatus:

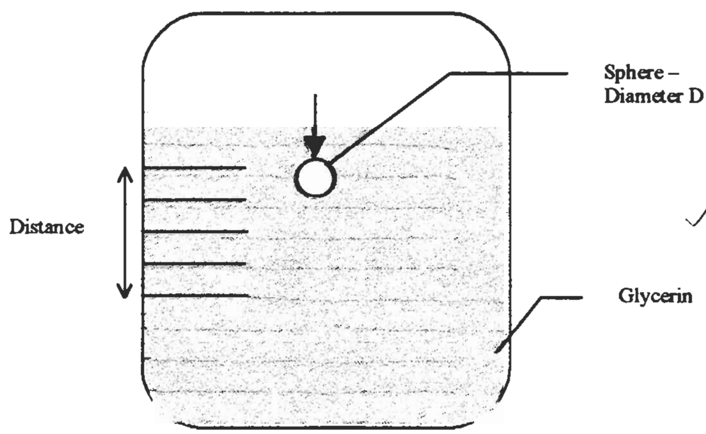


Figure 2 – Graduated cylinder filled with glycerin.

Theory:

$Re = UD\rho/\mu$

- Re = Reynolds number (dimensionless quantity)
- U = speed of sphere falling through the fluid
- D = sphere diameter
- ρ = fluid density
- μ = dynamic viscosity coefficient

Navier-Stokes Equation Approximation when Re$\ll$1: Good !

$$\nabla P = \mu \nabla^2 \mathbf{u}$$

Continuity Equation:

$$\nabla \mathbf{u} = 0$$

Combining the Navier-Stokes Approximation and the Continuity Equation:

$$F = 6\pi\mu r U$$

- F = drag force on the falling sphere
- r = sphere radius

Setting the sum of the forces equal to zero and solving for μ:

$$\mu = (2/9)(gr^2/U)(\rho' - \rho)$$

- g = gravitational constant
- ρ' = sphere density

Procedure:

The Reynolds number is a dimensionless ratio of the inertial forces to the viscous forces. When Re $\ll$ 1, the inertial forces are insignificant with respect to the viscous forces and the flow is referred to as "Stokes" flow. This condition allows the Navier-Stokes equation to be simplified significantly. Combining this approximated form of the Navier-Stokes equation with the Continuity equation yields a relationship between the drag force on the falling sphere and the dynamic viscosity. Since the sphere falls through the fluid at terminal velocity (does not accelerate) the forces on the sphere sum to zero. The weight of the sphere acts downward against the drag and buoyancy forces. The dynamic viscosity coefficient can then be found by rearranging the summed force equation.

The apparatus for this section can be seen in Figure 2. It consists of a graduated cylinder filled with glycerin. A metal sphere of given diameter is dropped into the fluid and timed as it falls a determined distance at terminal velocity. Metal spheres of three different radii were used and three sets of data were collected for each sphere size. The raw data can be found in Appendix A.

Apparatus:

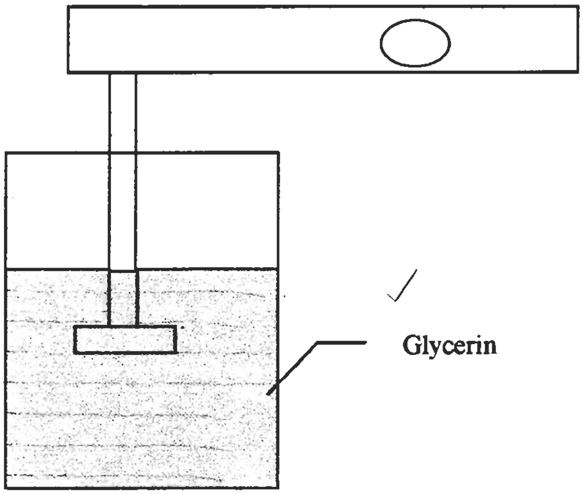


Figure 3 – Viscosity meter in beaker of glycerin.

Procedure:

The viscosity meter (see Figure 3) is pre-calibrated to measure the viscosity of a fluid with a turning spindle. The shear stress of the fluid applies a torque to the turning spindle that is directly proportional to the viscosity of the fluid (Newton's law of linear shear stress in a fluid). By observing the relative twist between the motor drive and the spindle on a calibrated dial, the associated viscosity of the fluid can be looked up on a chart that comes with the viscosity meter. Several readings were taken at various spindle speeds. The raw data can be found in Appendix A.

Part 2 – Laminar/Turbulent Pipe Flow

Apparatus:

(Similar to apparatus used in Part 1, section (i). A glycerin-water mixture is used instead of pure glycerin. See Figure 1)

Theory:

ρ = now equals the density of a glycerin-water mixture, which is lower than the density of pure glycerin

Laminar Regime:

$$\Delta P \propto Q$$

$$\mu = k(\pi/8)(R^4/L)$$

See Part 1 section (i) for an explanation of the variables

Turbulent Regime:

$\Delta P \propto Q^{1.75}$

$\Delta P \propto Re^{1.75}$

$Re = (4/\pi)(\rho Q/\mu D)$

See Part 1, section (i) for an explanation of the variables

Procedure:

For this section, a mixture of glycerin and water was used instead of pure glycerin alone. This reduces the density of the fluid and the viscosity and reduces the amount of time between data collection. When the flow rate is slow and laminar, the results of this section should be very similar to the results found in Part 1 section (i). As the flow rate increases, however, the Reynolds number increases and the flow becomes turbulent. At high Reynolds number, the inertial forces become proportional to the viscous forces and the viscous forces can no longer dampen out small disturbances in the fluid. The fluid begins to mix as it flows down the pipe and a new relationship develops between the pressure difference along a length of the pipe and the fluid flow rate through the pipe.

The apparatus and experiment procedure is similar to that of Part 1 section (i). Data were recorded at various flow rates that corresponded to both laminar and turbulent flow. The raw data can be found in Appendix A.

Results and Discussion

Viscosity Measurement:

Laminar Pipe Flow	Sphere in Stokes Flow	Viscosity Meter
$\mu_{avg} = 0.755$ kg/m/s	$\mu_{avg} = 0.964$ kg/m/s	$\mu_{avg} = 0.807$ kg/m/s

These results show that the viscosity of the glycerin is probably somewhere between 0.7 and 1 kg/m/s. If the experiments had been performed without any error, then the laminar pipe flow measurement would have been the most accurate. This is because the equations for the dynamic viscosity coefficient in the laminar pipe flow case were exact solutions derived from the Navier-Stokes equation of motion. They should give exact results as long as there is no experimental error and as long as the flow in the pipe is laminar. A graph of the pressure change versus the flow rate for the pipe flow (see graph in Appendix B) shows that the flow is indeed very laminar, since the data were highly linear ($R^2 = 0.998$). The equations for the sphere in Stokes flow were derived from approximations of the Navier-Stokes equation assuming that the Reynolds number is much less than one. A very low Reynolds number means that the inertial forces in the fluid are much smaller than the viscous forces. If we assume that the inertial forces are zero, then all of the drag on the sphere falling in the fluid is due to the viscous forces. This assumption greatly simplifies the Navier-Stokes equation, but is only an approximation since inertial forces will always be present to a certain extent. The results from the Stokes flow can never be perfectly accurate, even if there are no experimental errors. The accuracy of the viscosity meter could potentially be very high, but since it is based on how well it was calibrated, we cannot assume that it is the most accurate since we do not know the details of its calibration. Even without experimental errors, the accuracy of the viscosity meter still depends on the calibration, giving even stronger evidence that the laminar pipe flow experiment would be the most accurate if there were no experimental errors.

Good!

The laminar pipe flow experiment would give the most accurate results with no experimental error, but since experimental error is always present, we cannot just assume that the results for that experiment were the most accurate. The results from the sphere in Stokes flow are based on equations that are only approximations, but these approximations become more and more accurate as the Reynolds number decreases. The viscosity coefficient found from the Stokes flow and given in the table above is an average of the values found from the different sized spheres. This is not a good indicator of the results found from this section. The Reynolds number is directly proportional to both the diameter of the sphere and the velocity at which it falls through the fluid. As the diameter of the sphere decreases, the velocity also decreases, which means the Reynolds number goes down considerably as the sphere size gets smaller. Therefore the results found with the smallest sphere should be the most accurate and averaging the values over all the sphere sizes only decreases the accuracy of the results. A better indicator of the Stokes flow results would be to report the values found with the smallest sphere. Even at this size, however, the Reynolds number is 0.02 (see Appendix B). This is a small value, but still close enough to 1.0 to contain inaccuracy.

Very Good!

An even better way to approximate the viscosity coefficient based on the data found in the Stokes flow experiment is to determine a relationship between the approximate viscosity coefficient and the Reynolds number and extrapolate a value of the coefficient at zero Reynolds number. The viscosity values at each sphere size can be plotted against the estimated Reynolds number for that sphere size and a curve can be fit to that data to estimate the relationship between the two. Since there were only three sphere sizes used in this experiment giving three data points, the only polynomial curves that can be fit to the data are a linear or a quadratic curve. Flow theory can give us an indication of which fit should be better. The Stokes-Oseen formula gives a more accurate indication of drag on a sphere for very small, but not completely negligible Reynolds numbers. Re-deriving the equations for the viscosity using the Stokes-Oseen formula gives an equation similar to the one found using the Stokes formula except an additional term is subtracted. This additional term is a function of the Reynolds number and indicates that the equations we used to calculate the viscosity will always give a higher value than is actually correct. The

Please note that it is important to show this new derivation.

The second (new) term should be $\frac{3}{16}$ Re!

Please see comments at the end.

difference between the drag forces using the Stokes-Oseen formula and the Stokes formula for the smallest sphere size is only about %0.38, which is very small, but indicates that the viscosity can be predicted more accurately. As the Reynolds number goes to zero, the Stokes-Oseen predictions and the Stokes predictions should converge to the same value. A plot of both the Stokes predictions and the Stokes-Oseen predictions can be found in Appendix B. When a quadratic curve is fit to each data set, the Stokes predictions extrapolate to a value of 0.9252 kg/m/s and the Stokes-Oseen predictions (the "modified" series on the graph) extrapolate to a value of 0.9255 kg/m/s. These values are extremely close, indicating that the value of viscosity is probably about 0.925 kg/m/s and show that a quadratic fit to the data set was probably the best choice.

Very interesting!

Best estimation of μ: 0.925 kg/m/s

In my opinion, the Stokes flow method of calculating the viscosity coefficient is better than the laminar pipe flow method. Even though the Stokes flow method is based on approximations and the laminar pipe flow equations are exact, the Stokes flow method can be used to determine the viscosity coefficient more accurately. The accuracy of the laminar pipe flow method will always be reduced due to high experimental error. Many more variables are observed during a single run of the pipe flow method than a single run of the Stokes method, which increases the chance that data will be indicated or measured inaccurately. Roughness of the pipes, edge effects of the valves and manometers, inaccuracies in the manometer, and the number of variables observed all contributed to the experimental errors in the pipe flow method. The only variable being measured in the Stokes flow method is the time it takes the sphere to fall a certain distance. Human error in running the stopwatch is the main source of experimental error and that decreases as the amount of time the sphere falls increases. The Stokes flow method is simpler, more repeatable, and with the extrapolation described in the previous paragraph, probably more accurate than the pipe flow method.

Very Good!

Laminar/Turbulent Transition:

The graphs of the pressure gradient versus the flow rate and the pressure gradient versus the Reynolds number found in Appendix B show that at a certain point on each graph, the relationships between the variables changes dramatically. The data goes from a linear relationship, to a power relationship with a best fit power of 1.77. Theory predicts that the pressure gradient will change linearly with both the flow rate and the Reynolds number up to a point, and then vary by a power relationship with a power of 1.75. This point is in the transition range from laminar flow to turbulent flow and occurs at the critical Reynolds value. The flow profile in laminar flow is parabolic; maximum flow rate in the center of the pipe and zero flow rate at the walls of the pipe. The parabolic curve results from the ordered flow within the pipe. The viscous forces are more significant than the inertial forces since they Reynolds number is low and the viscous forces tend to dampen out small fluctuations in the flow. As the Reynolds number increases, however, small flow fluctuations are not dampened out and the fluid tends to start mixing. This mixing flattens the flow profile and fluid a small distance from the center of the pipe travels at the same speed as fluid at the center of the pipe. This means that for a given flow rate, the velocity gradient at the wall of the turbulent flow is much greater than the velocity gradient at the wall of the laminar flow, since the velocity changes from its maximum value to zero over a shorter distance. Newton's law of viscosity tells us that the shear stress in a fluid is directly proportional to the velocity gradient for a Newtonian fluid, which we assume to be the case for glycerin. At higher velocity gradients, the shear stress increases. The higher shear stress in the turbulent flow results in a greater pressure drop over a given length of pipe than for the laminar flow. This is why the pressure gradient increases significantly more as the flow rate goes up in the turbulent regime than in the laminar regime.

Very Good!

Excellent!

In our experiment, we found the critical Reynolds number, or the Reynolds number at which the flow shifts from laminar to turbulent, to be about 2000 (see graph of ΔP vs. Re in Appendix B). This is a relatively low value compared to many modern estimations. This is due to flaws and disturbances in our experiment. The flow shifts from laminar to turbulent when the viscous forces can no longer dampen out

Please note that the explanation is great, but it is important to include sketches and equations here. See comments at the end.

the small disturbances in the flow. The more disturbances there are in the flow, the sooner the flow will shift to turbulence. There are many sources of disturbances in our experiment, including edge effects of the valves and manometers, vibrations do to the surrounding environment, roughness in the pipe, and any contaminants in the fluid. Reducing these disturbances would have increased our critical Reynolds number.

✓ Very Good!

Very Good Report! Fantastic presentation and good clear writing!

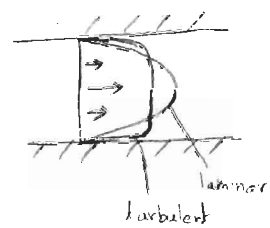
94%

However, please note the following points:

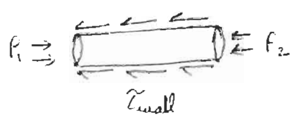
- It is important to include sketches in explanations (like why Δp increases in turbulent flow).

  Important things to improve the explanation are

  - sketch of the velocity profiles.
  - show relationship between Δp and τ_{wall} from momentum balance



velocity profiles.



$$(P_1 - P_2) A = \tau_{wall} (\pi D L).$$

$$\Rightarrow \boxed{\Delta P = \tau_{wall} \left(\frac{\pi D L}{A}\right)}$$

- Stokes - Oseen drag.

  Using it in the force balance gives.

$$6\pi \mu R U + \frac{9}{4}\pi \rho R^2 U^2 = \frac{4}{3}\pi(\rho' - \rho) R^3 g .$$

$$\Rightarrow \mu = \frac{2}{9}\frac{gR^2}{U}(\rho' - \rho) - \frac{3}{8}\rho R U$$

$$\Rightarrow \mu = \mu_{predicted} - \left(\frac{3}{16} Re\right)\mu .$$

$$\Rightarrow \mu_{predicted} = \mu\left(1 + \frac{3}{16} Re\right).$$ $\Rightarrow$ A nice functional form for the relationship.

Section 1(I) - Viscosity From Pipe Flow Experiment

run	h_1 (cm)	h_2 (cm)	volume (mL)	time (s)
1	22.5	7.9	10	21.32
2	38	13.4	10	12.75
3	46.8	16.6	10	9.97
4	57.1	20.4	10	8.38
5	66.5	23.4	10	7.03

Section 1(ii) - Sphere in Stokes flow

run	dia. (in)	dist. (cm)	time (s)
1	0.125	30	8.28
2	0.125	30	8.22
3	0.125	30	8.35
4	0.09375	30	14.41
5	0.09375	30	14.31
6	0.09375	30	14.34
7	0.0625	30	31.47
8	0.0625	30	31.16
9	0.0625	30	31.75

Section 1(iii) - Viscosity Meter

speed	% scale	factor
6	16.0%	50
6	15.5%	50
6	16.0%	50
12	32.2%	25
12	32.5%	25
12	32.5%	25
30	82.5%	10
30	82.0%	10
30	81.5%	10
60	---	5
60	---	5
60	---	5

Raw Data

Laminar/Turbulent Pipe Flow

run	h_1 (cm)	h_2 (cm)	vol. (mL)	time (s)	condition
1	65.3	22.7	2000	15.53	rough
2	61.3	22.7	2000	17.97	rough
3	56.3	23.1	2000	19.12	rough
4	51.6	23.3	1000	10.53	rough
5	46.5	23.7	1000	11.84	transition
6	44	24	1000	12.16	transition
7	41.2	24.4	1000	13.16	transition
8	38	24.4	1000	15.47	transition
9	36	24.2	1000	17.03	towards laminar
10	33.8	24	1000	18.65	towards laminar
11	32.2	23.9	400	8.59	nearly laminar
12	30.5	23.8	400	10.44	nearly laminar
13	28.9	23.7	400	13.41	completely laminar
14	27.7	23.6	300	13.34	completely laminar
15	26.4	23.5	200	13.38	completely laminar
16	25.3	23.5	100	12.66	completely laminar

Appendix B

FM1 Tabulated Data

Part 1 - Viscosity Measurement

Section 1(i) - Viscosity From Pipe Flow Experiment

ρ =	1260 kg/m^3
g =	9.81 m/s^2
R =	4.75E-03 m
L =	1 m

Ex. Calculations:

$\Delta P = \rho g(h_1 - h_2)$

1804.6 = 1260*(9.81)*(.225-.079)

Q = volume/time

469.04E-09 = (10E-06)/(21.32)

μ avg. = $k(\pi/8)(R^4/L)$ k = $\Delta P/\Delta Q$

0.755 = (3.775E-09)*(π/8)*(0.00475^4/1)

run	h_1 (m)	h_2 (m)	$h_1 - h_2$ (m)	Δ P (Pa)	vol. (m^3)	time (s)	Q (m^3s^{-1})
1	0.225	0.079	0.146	1804.6	10.E-06	21.32	469.04E-09
2	0.38	0.134	0.246	3040.7	10.E-06	12.75	784.31E-09
3	0.468	0.166	0.302	3732.9	10.E-06	9.97	1.E-06
4	0.571	0.204	0.367	4536.3	10.E-06	8.38	1.19E-06
5	0.665	0.234	0.431	5327.4	10.E-06	7.03	1.42E-06

k = 3.775E+09 (from chart)

μ avg. = 0.755

Section 1(ii) - Sphere in Stokes flow

ρ =	1260 kg/m^3
ρ'=	7780 kg/m^3
g =	9.81 m/s^2

Ex. Calculations:

U = distance/time

0.036217 = (0.3)/(8.28)

$\mu = (2/9)(gR^2/U)(\rho' - \rho)$

0.989 = (2/9)(9.81*0.001588^2/(0.036217))*(7780-1260)

$\mu_{error} = -(3/8)\rho rU$

-0.027 = -(3/8)*(1260)*(0.001588)*(0.036217)

$\mu_{modified} = \mu_{pred} + \mu_{error}$

0.962 = 0.989 - 0.027

% drag force error = $(3/8)(\rho/\mu)RU$

.0275 = (3/8)*(1260/0.989)*(0.001588)*(0.03622)

Fantastic effort!

run	r (m)	dist. (m)	time (s)
1	1.588E-03	0.3	8.28
2	1.588E-03	0.3	8.22
3	1.588E-03	0.3	8.35
4	1.191E-03	0.3	14.41
5	1.191E-03	0.3	14.31
6	1.191E-03	0.3	14.34
7	793.75E-06	0.3	31.47
8	793.75E-06	0.3	31.16
9	793.75E-06	0.3	31.75

avg. time (s)	U (m,/s)	μ_{pred}	Re
8.28	0.0362173	0.989	0.146
14.35	0.0209011	0.964	0.065
31.46	0.0095359	0.939	0.020

μ avg. =	0.964

Re	% $drag_{error}$
0.146	2.75%
0.065	1.22%
0.02	0.38%

Re	μ_{error}	$\mu_{modified}$
0.146	-0.027	0.962
0.065	-0.0117583	0.952
0.02	-0.0035764	0.936

Better approximation of μ: 0.925 (from chart)

Section 1(iii) - Viscosity Meter

Ex. Calculations:

μ_{cent} = (% scale)(factor)

8 = 0.16*50

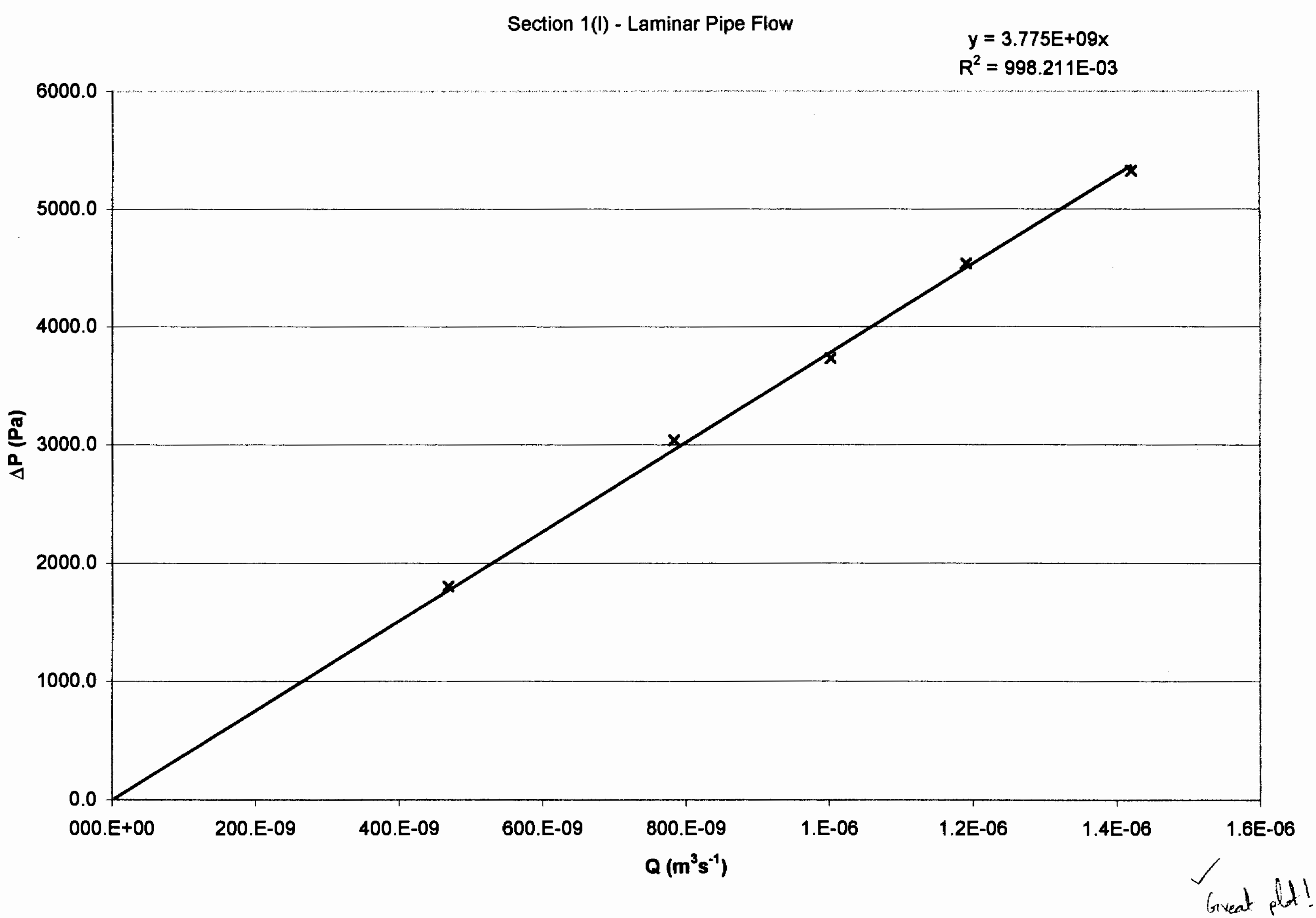
$\mu = 10\% \mu_{cent}$

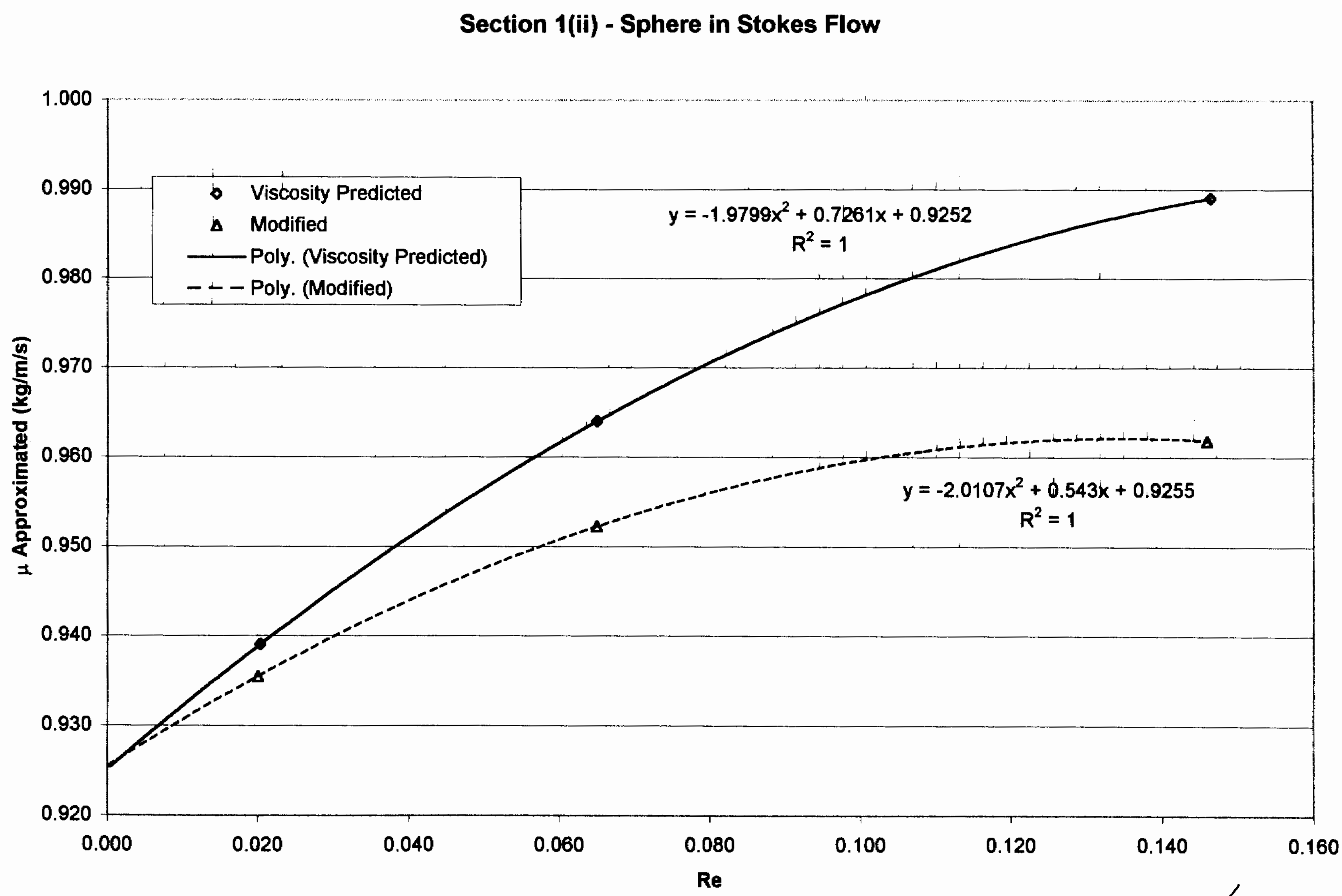
0.8 = 0.1*8

speed	% scale	factor	μ_{cent}	μ
6	16.0%	50	8	0.8
6	15.5%	50	7.75	0.775
6	16.0%	50	8	0.8
12	32.2%	25	8.05	0.805
12	32.5%	25	8.125	0.8125
12	32.5%	25	8.125	0.8125
30	82.5%	10	8.25	0.825
30	82.0%	10	8.2	0.82
30	81.5%	10	8.15	0.815
60	---	5	---	---
60	---	5	---	---
60	---	5	---	---

μ avg. =	0.807

Good Work


Section 1(l) - Laminar Pipe Flow
y = 3.775E+09x
R2 = 998.211E-03
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
ΔP (Pa)
000.E+00
200.E-09
400.E-09
600.E-09
800.E-09
1.E-06
1.2E-06
1.4E-06
1.6E-06
Q (m3s-1)
Great plot!


Section 1(ii) - Sphere in Stokes Flow
Viscosity Predicted
Modified
Poly. (Viscosity Predicted)
Poly. (Modified)
y = -1.9799x^2 + 0.7261x + 0.9252
R^2 = 1
y = -2.0107x^2 + 0.543x + 0.9255
R^2 = 1
μ Approximated (kg/m/s)
1.000
0.990
0.980
0.970
0.960
0.950
0.940
0.930
0.920
0.000
0.020
0.040
0.060
0.080
0.100
0.120
0.140
0.160
Re

FM1 Tabulated Data

Part 2 - Laminar/Turbulent Pipe Flow

ρ =	1097 kg/m^3
R =	0.00475 m

Ex. Calculations:

ΔP (see previous results)

Q (see previous results)

μ (see previous results)

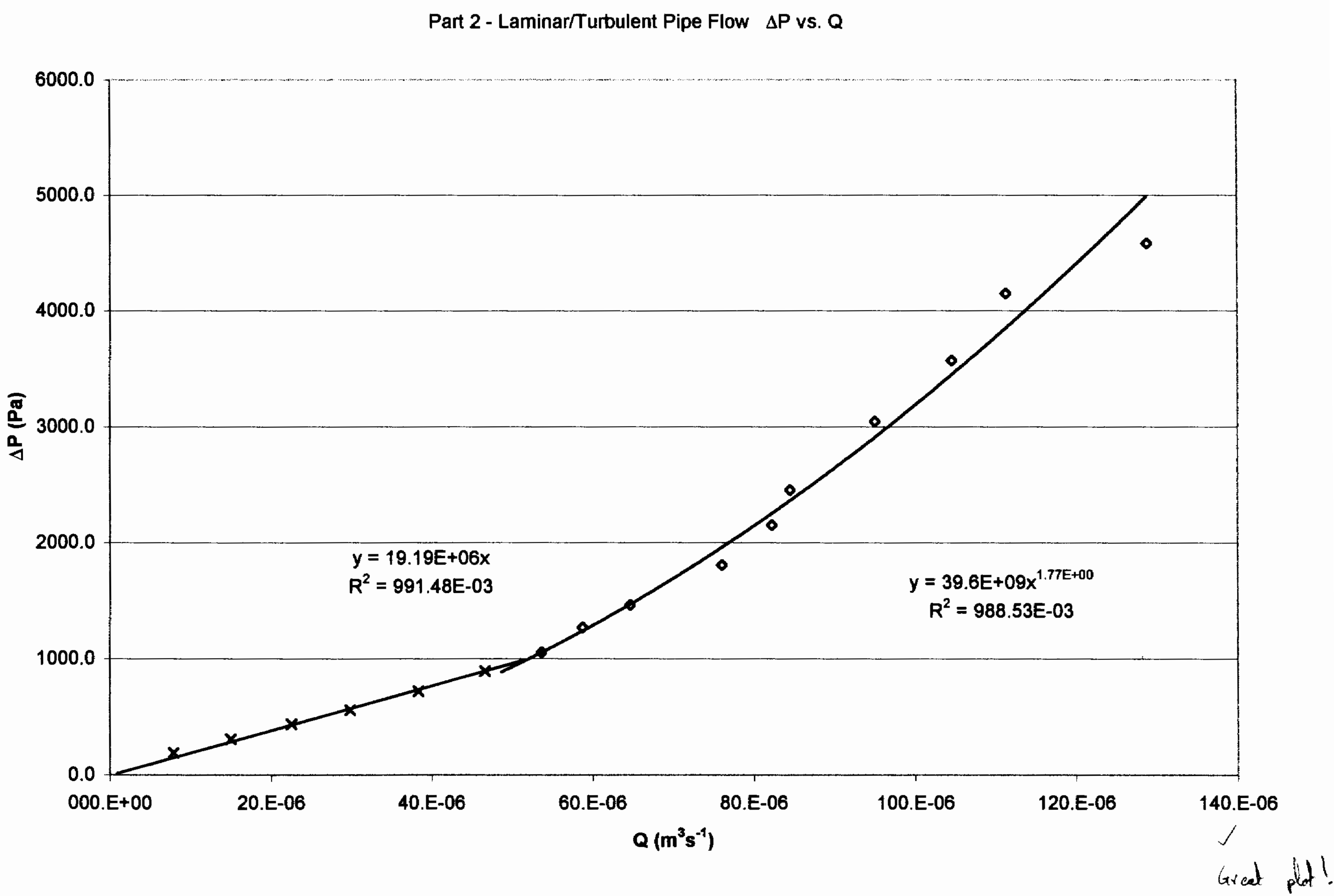
Re = $(4/\pi)(\rho Q/\mu D)$

4935.6 = $(4/\pi)$*(128.8E-06)*(1097)/((0.004)*(2*0.00475))

run	h_1 (m)	h_2 (m)	$h_1 - h_2$ (m)	ΔP (Pa)	vol. (m^3)	time (s)	Q (m^{3}s^{-1})	Re
1	0.653	0.227	0.426	4584.4	2.E-03	15.5	128.8E-06	4935.6
2	0.613	0.227	0.386	4154.0	2.E-03	18.0	111.3E-06	4265.5
3	0.563	0.231	0.332	3572.8	2.E-03	19.1	104.6E-06	4008.9
4	0.516	0.233	0.283	3045.5	1.E-03	10.5	95.E-06	3639.6
5	0.465	0.237	0.228	2453.6	1.E-03	11.8	84.5E-06	3236.9
6	0.44	0.24	0.2	2152.3	1.E-03	12.2	82.2E-06	3151.7
7	0.412	0.244	0.168	1807.9	1.E-03	13.2	76.E-06	2912.2
8	0.38	0.244	0.136	1463.6	1.E-03	15.5	64.6E-06	2477.4
9	0.36	0.242	0.118	1269.9	1.E-03	17.0	58.7E-06	2250.4
10	0.338	0.24	0.098	1054.6	1.E-03	18.7	53.6E-06	2055.0
11	0.322	0.239	0.083	893.2	400.E-06	8.6	46.6E-06	1784.6
12	0.305	0.238	0.067	721.0	400.E-06	10.4	38.3E-06	1468.4
13	0.289	0.237	0.052	559.6	400.E-06	13.4	29.8E-06	1143.2
14	0.277	0.236	0.041	441.2	300.E-06	13.3	22.5E-06	861.9
15	0.264	0.235	0.029	312.1	200.E-06	13.4	14.9E-06	572.9
16	0.253	0.235	0.018	193.7	100.E-06	12.7	7.9E-06	302.7

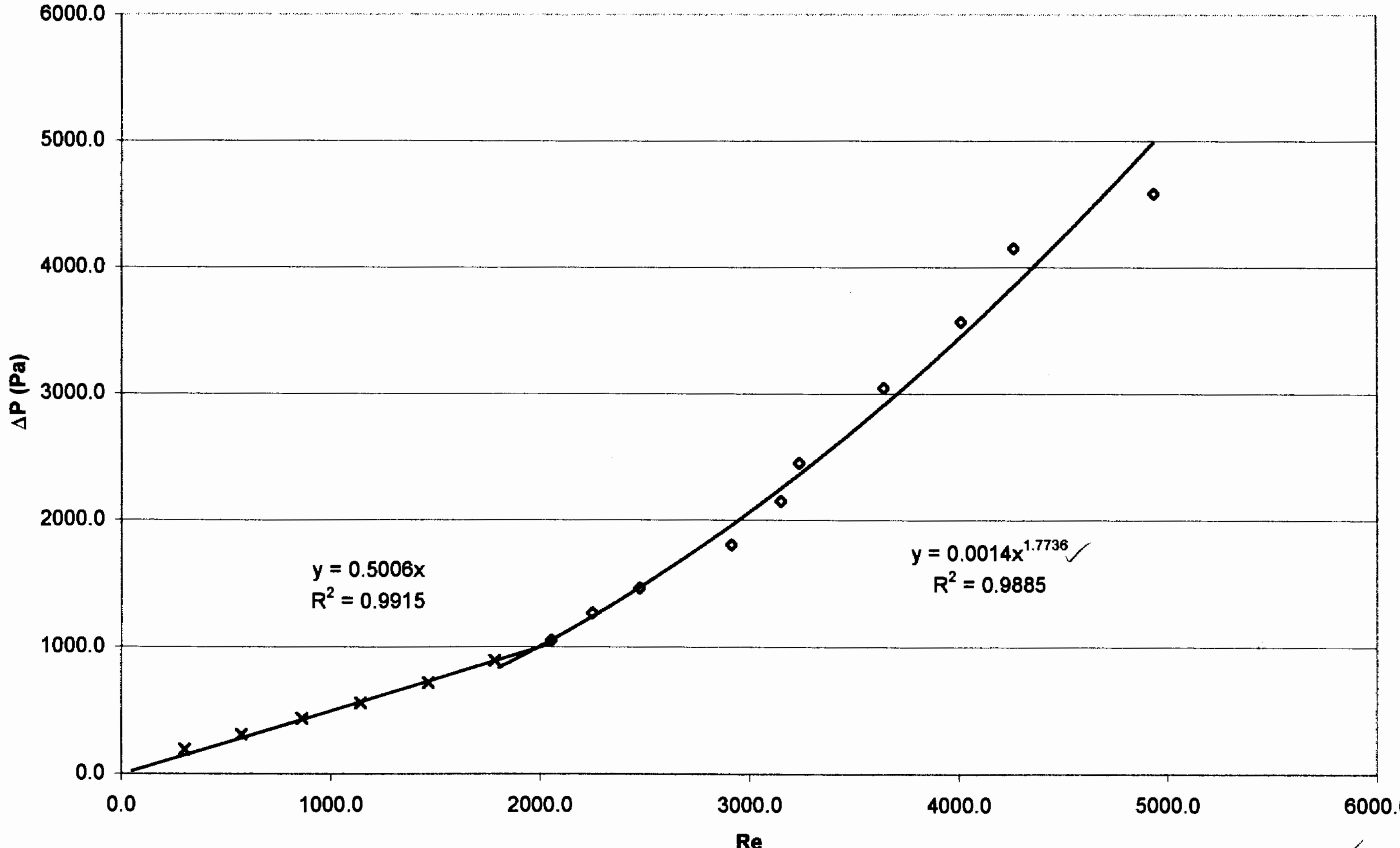
k = 19.19E+06 (from chart)

μ = 0.004



Part 2 - Laminar/Turbulent Pipe Flow ΔP vs. Q
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
ΔP (Pa)
y = 19.19E+06x
R2 = 991.48E-03
y = 39.6E+09x1.77E+00
R2 = 988.53E-03
000.E+00
20.E-06
40.E-06
60.E-06
80.E-06
100.E-06
120.E-06
140.E-06
Q (m3s-1)
Great plot!

Part 2 - Laminar/Turbulent Pipe Flow ΔP vs. Re



Good plot!